Magnetohydrodynamic Simulations of Black Hole Accretion

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Abstract. We discuss the results of three-dimensional magnetohydrodynamic simulations, using a pseudo-Newtonian potential, of thin disk ($h/r \approx 0.1$) accretion onto black holes. We find (i) that magnetic stresses persist within $r_{\rm ms}$, the marginally stable orbit, and (ii) that the importance of those stresses for the dynamics of the flow depends upon the strength of magnetic fields in the disk outside $r_{\rm ms}$. Strong disk magnetic fields ($\alpha \gtrsim 0.1$) lead to a gross violation of the zero-torque boundary condition at $r_{\rm ms}$, while weaker fields ($\alpha \sim 10^{-2}$) produce results more akin to traditional models for thin disk accretion onto black holes. Fluctuations in the magnetic field strength in the disk could lead to changes in the radiative efficiency of the flow on short timescales.

INTRODUCTION

There is now a consensus that, in well-ionized accretion disks, magnetorotational instabilities (MRI) create turbulence that provides the 'anomalous viscosity' required to drive accretion [1]. This removes a major uncertainty that afflicted previous theoretical models for black hole accretion, and opens the possibility of using numerical simulations to study directly the structure and variability of the accretion flow. Questions that we might hope to address include:

- The magnetohydrodynamics (MHD) of the flow as it crosses $r_{\rm ms}$, the marginally stable circular orbit. Recent work has suggested that MHD effects interior to $r_{\rm ms}$ (in the 'plunging' region) could invalidate existing models of black hole accretion, with consequences that include an increase in the predicted radiative efficiency of thin disk accretion [2–4]. This suggestion remains controversial [5].

TABLE 1. Summary of the simulations discussed in this article. All runs have the same sound speed, $c_s/v_\phi = 0.065$ (evaluated at $r_{\rm ms}$), spatial domain ($0.666 < r/r_{\rm ms} < 3.3$, $z/r_{\rm ms} = \pm 0.166$, $\Delta\phi = 45°$), and resolution ($n_r = 200$, $n_z = 40$, $n_\phi = 60$). The time units are such that the orbital period at the last stable orbit is $P = 7.7$.

Run	Vertical boundary conditions	Initial field	Final time	Output times
Azimuthal field	$B_z = v_z = 0$	$\beta_\phi = 100$	600	400-600
Vertical field	Periodic	$\beta_z = 5000$	250	100-250
High saturation	Periodic	$\beta_z = 500$	150	62.5-150

- The predicted variability in emission from the inner disk.
- The structure of the disk magnetic field and the rate of transport of magnetic flux through $r_{\rm ms}$, which together determine the efficiency of the Blandford-Znajek mechanism for extracting spin energy of the black hole [6–8].

This article presents results from simplified MHD simulations [9,10] of black hole accretion that focus on the first of these questions. We outline the numerical approach, and discuss our results and how they compare with those of other groups [11–13]. Our conclusion from work to date is that MHD effects in the plunging region *can* have an important influence on the dynamics of the inner disk flow, provided that the magnetic fields in the disk are already moderately strong.

NUMERICAL SIMULATIONS

Ideally, we would like to simulate a large volume of disk (to allow for global effects [14] and ease worries about treatment of the boundaries), for a long time period (to average out fluctuations), at high resolution (to resolve the most unstable scales of the magnetorotational instability), with the most realistic physics possible. Unfortunately, we can't, so compromises are needed. Our approach has been to simulate the simplest disk model that we believe includes the essential physical effects, while aiming for the highest resolution near and inside the last stable orbit.

We use the ZEUS MHD code [15,16] to solve the equations of ideal MHD within a restricted 'wedge' of disk in cylindrical (z, r, ϕ) geometry. The equation of state is isothermal, and a Paczynski-Wiita potential [17] is used to model the effect of a last stable orbit within the Newtonian hydrocode. To further simplify the problem, we ignore the vertical component of gravity and consider an unstratified disk model. The boundary conditions are set to outflow at the radial boundaries, are periodic in azimuth, and are either periodic or reflecting in z. The simulations begin with a stable, approximately Gaussian surface density profile outside $r_{\rm ms}$, which is threaded with a weak magnetic field. This initial seed field has a constant ratio of thermal to magnetic energy β, and is either vertical or azimuthal. We evolve this setup, which is immediately unstable to the MRI, until a significant fraction of the mass has been accreted, and plot results from timeslices towards the end of the runs when the magnetic fields have reached a saturated state.

Table 1 summarizes the parameters of three simulations, which are improved versions of those previously reported [9]. The simulated disks are 'thin' in the sense that pressure gradients are negligibly small in the disk outside $r_{\rm ms}$. Specifically, the ratio of sound speed to orbital velocity, which in a stratified disk is approximately equal to h/r, is $\lesssim 0.1$ at the last stable orbit. We have not (yet) attempted the more difficult task of simulating very thin disks, which have longer viscous timescales, and caution against extrapolating our conclusions into that regime.

Following a suggestion by Charles Gammie (personal communication), we investigated whether varying the magnetic field strength in the *disk* (i.e. at $r > r_{\rm ms}$) led to changes in the dynamics of the flow within the plunging region. To vary the field strength in the simulations, we make use of the fact that the saturation level of the MRI can be altered by varying the net flux of seed fields in the initial conditions. Local simulations [18] show that the resultant Shakura-Sunyaev α parameter [19] is approximately,

$$\alpha \sim 10^{-2} + 4 \frac{\langle v_{Az} \rangle}{c_s} + \frac{1}{4} \frac{\langle v_{A\phi} \rangle}{c_s} \tag{1}$$

where v_{Az} and $v_{A\phi}$ are the Alfven speeds for initial conditions with uniform vertical and azimuthal seed fields, and c_s is the sound speed. For our simulations, we find that the range of initial conditions (and vertical boundary conditions) shown in Table 1 leads to a variation in α between 10^{-2} and 10^{-1} in the disk.

Of course, this is a numerical trick. The *true* value of α in the disk immediately outside $r_{\rm ms}$ will probably depend upon details of the disk physics (for example, the relative importance of gas and radiation pressure), and may vary with time.

DYNAMICS OF THE FLOW CROSSING $R_{\rm MS}$

Figure 1 illustrates the geometry of the simulations. All MHD disk simulations look broadly similar, and these are no exception. We obtain a pattern of surface density fluctuations that are strongly sheared by the differential rotation, and disk magnetic fields that are predominantly azimuthal. A map of the ratio of magnetic to thermal energy, also shown in the Figure, displays clearly the predicted increase [2] in the relative importance of magnetic fields

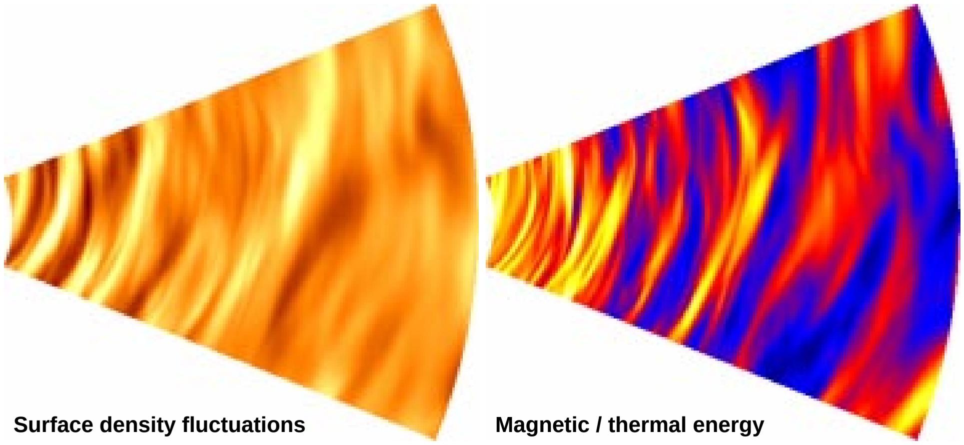

FIGURE 1. Maps showing (left panel) the surface density fluctuations and (right panel) the ratio of the energy density in magnetic fields to the thermal energy, from a simulation in which the saturation value of the magnetorotational instability in the disk was artificially boosted. The spatial domain covers $0.666 < r/r_{\rm ms} < 3.3$. A clear increase in the relative importance of magnetic fields in the inner regions of the disk, and within the marginally stable orbit, is obtained.

near and interior to the marginally stable orbit. Determining the influence of these fields upon the dynamics of the flow in the inner disk and in the plunging region is the main goal of the calculations.

Figure 2 shows the magnetic torque as a function of radius in the three simulations, expressed as the magnetic contribution to the equivalent Shakura-Sunyaev α parameter,

$$\alpha_{\rm mag} = \frac{2}{3} \langle \frac{-B_r B_\phi}{4\pi \rho c_s^2} \rangle . \tag{2}$$

There are also hydrodynamic (Reynolds) stresses, which are somewhat harder to measure [13], but which are found to be substantially smaller than the magnetic stresses in local simulations.

The three choices for the initial flux and vertical boundary conditions lead to large variations in the saturation level of the MRI and associated $\alpha_{\rm mag}$. The run with an initially azimuthal field produces an $\alpha_{\rm mag} \sim 10^{-2}$, while the run with a relatively strong initial vertical flux leads to a disk $\alpha_{\rm mag}$ that exceeds 0.1. This increase is qualitatively in agreement with local simulations [18]. For the run with the strongest field, the magnetic field energy density in the disk near $r_{\rm ms}$ is on average near equipartition with the thermal energy ($\beta \sim 1$). There are large fluctuations with time, however, including brief periods where the magnetic energy substantially exceeds the thermal energy. In all runs, the relative importance of magnetic fields compared to the thermal energy increases within $r_{\rm ms}$.

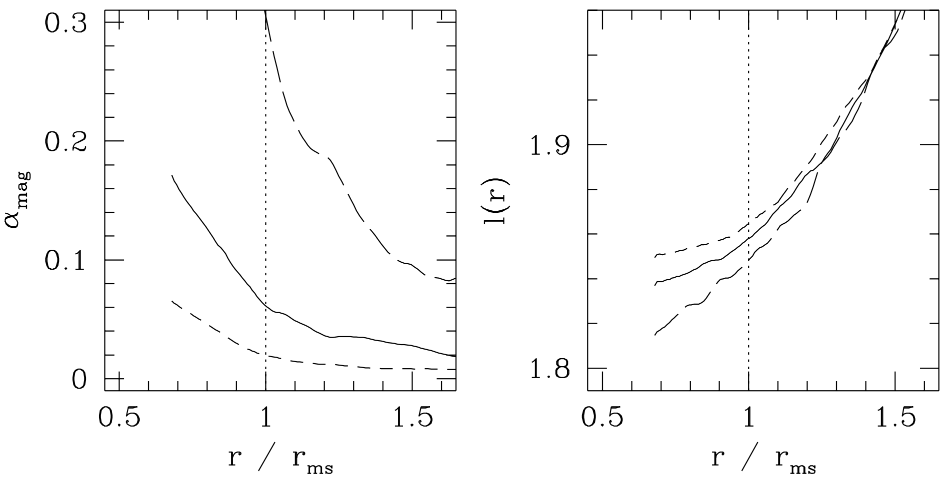

FIGURE 2. The dynamics of the flow inside the marginally stable orbit is correlated with the magnetic field strength in the disk. The left panel shows the contribution made by magnetic stresses to the Shakura-Sunyaev α parameter as a function of radius for the three runs: azimuthal initial field (short dashes), vertical initial field (solid), and strong vertical initial field (long dashes). The right panel shows the resulting specific angular momentum of the flow. All results are averaged over several independent timeslices to reduce fluctuations. Strong disk fields lead to a larger violation of the zero-torque boundary condition at $r_{\rm ms}$.

Figure 2 also shows how the torques influence the dynamics of the flow. We plot the specific angular momentum of the flow l as a function of radius. Hydrodynamic models for thin disk accretion onto black holes obtain $dl/dr = 0$ within the last stable orbit, corresponding to a zero-torque boundary condition for the disk [20]. A non-zero dl/dr implies transport of angular momentum (and implicitly energy) into the disk from within the plunging region. We find that dl/dr in the plunging region correlates with $\alpha_{\rm mag}$ in the disk. Weak disk fields lead to a small (but significant) decline in l within the last stable orbit, while strong fields lead to a steeply declining specific angular momentum profile at small radii. The former behavior is similar to that seen in our earlier simulations [9] (which also had rather weak fields), while the latter is comparable to the results obtained by Hawley [11] and Hawley and Krolik [12]. Their global simulations, which are substantially more ambitious than ours in terms of the included physics and spatial domain, did indeed generate relatively strong magnetic fields.

DISCUSSION

The limitations of the current simulations are myriad and obvious. We are accutely aware that effects in the disk corona could be important [21,22], and that there is more to General Relativity than a pseudo-Newtonian potential [23,24]. Nonetheless, we believe that some conclusions can be drawn from existing work:

- Unstratified global simulations confirm that magnetic torques persist within the last stable orbit, but suggest that if $\alpha \sim 10^{-2}$ their influence on the dynamics of the flow is relatively modest [9,13]. By modest we mean that the gradient of the specific angular momentum, dl/dr, is non-zero but small at and inside $r_{\rm ms}$. If these simulations reflect reality, existing models of black hole accretion would be a pretty good approximation for thin disks [5].
- A gross violation of the zero-torque boundary condition at $r_{\rm ms}$ is also possible [11,12]. This would increase the radiative efficiency of thin disk accretion above the usual $\epsilon \approx 0.1$, and have other consequences [2–4]. We believe that these strong effects *only occur* if $\alpha \gtrsim 0.1$ in the disk. This would be consistent with existing simulations [9,11], and with the results presented here.

For observations, these results suggest that the radiative efficiency of thin disk accretion may vary, both between systems, and in an individual system if the magnetic field strength in the disk varies with time.

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